Physics-Informed Neural Networks: A Rapid Solution of Structural Engineering Partial Differential Equations

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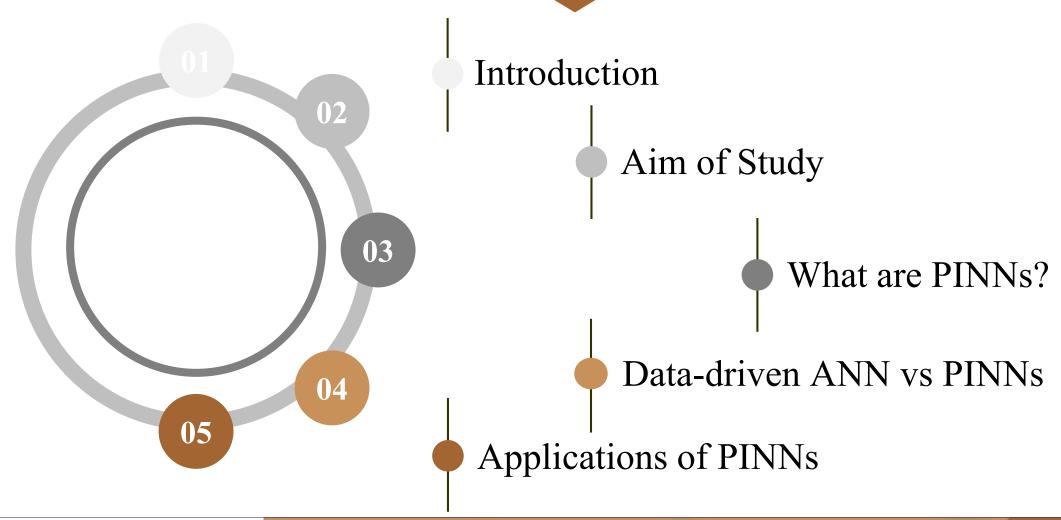








OUTLINE

























INTRODUCTION

- Nowadays, artificial neural networks (ANNs) are widely utilized in a broad range of engineering sectors thanks to the abundance of digital data, growing computing power, and advanced algorithms.
- A neural network as a data simulation method is considered a significantly valuable tool to approximate governing functions that map from the inputs to the outputs of a specific dataset.
- Recently, this technique was applied "*like a knight in shining armor*" to tackle complicated problems in structural engineering, such as predicting concrete properties, evaluating the behavior of concrete and steel structures, conducting seismic vulnerability assessments, identifying structural damage, developing adaptive optimal control approaches, etc.
- ✓ Indeed, the general impression about ANNs is that they can save computational efforts required in long processes of trials and errors by providing rapid data-driven closed-form models generated from correlations that map between the inputs and outputs of the training datasets.

















INTRODUCTION

- ✓ On the other hand, *purely data-driven* neural networks lack the robustness needed to accurately infer results in small data regimes or cases with strong nonlinearities and high dimensionality.
- Recently, a new approach to an artificial neural network, known as "physics-informed neural network", was introduced to the literature as a way of solving supervised learning tasks while respecting any given laws of physics.
- The main importance of this method arises from its ability to distill the mechanisms that govern the data evolution.
- As a result, the neural network's loss function is deeply embedded with the physics laws to constrain the training phase of the model to a feasible solution that can better represent the engineering issue as well as its data dependency (produces data-efficient models).



















AIM OF STUDY

This study aims to briefly describe and summarize recent applications of PINNs in the field of structural engineering with an emphasis on its capabilities in solving problems involving partial differential equations.

















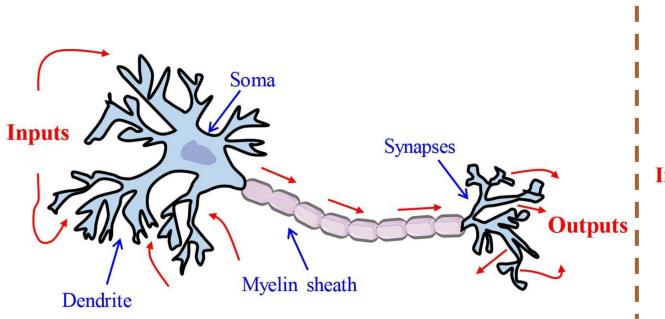


Figure 1: Illustration of a biological neuron

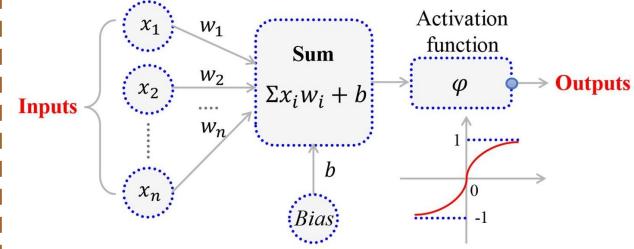


Figure 2: Illustration of an artificial neuron













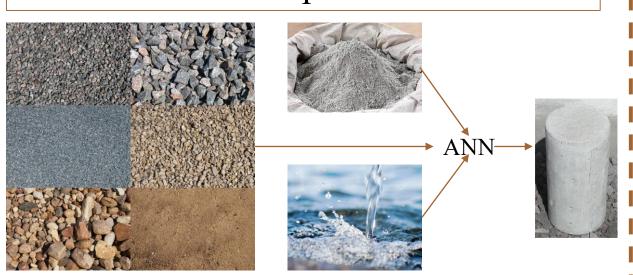




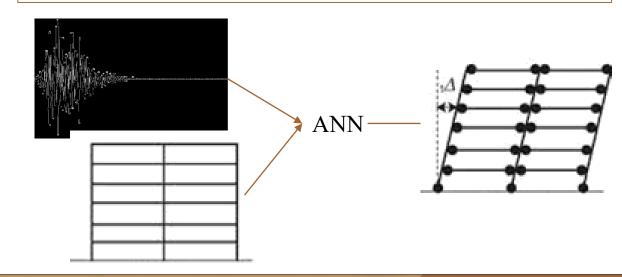


Applications of ANN

Develop approximate solutions to simulate complex relations



Developing efficient alternatives to computationally challenging problems















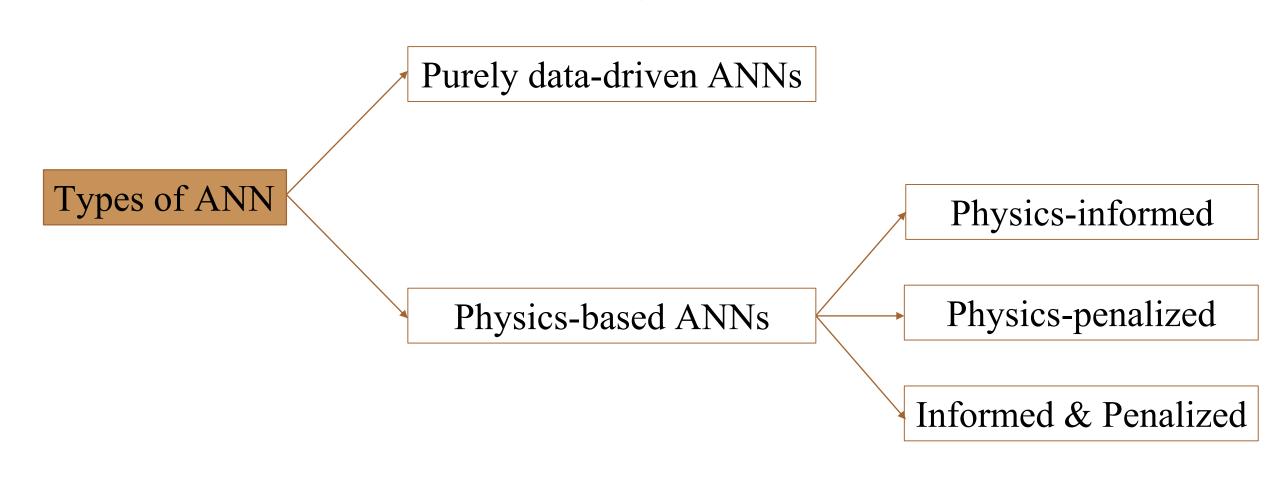
































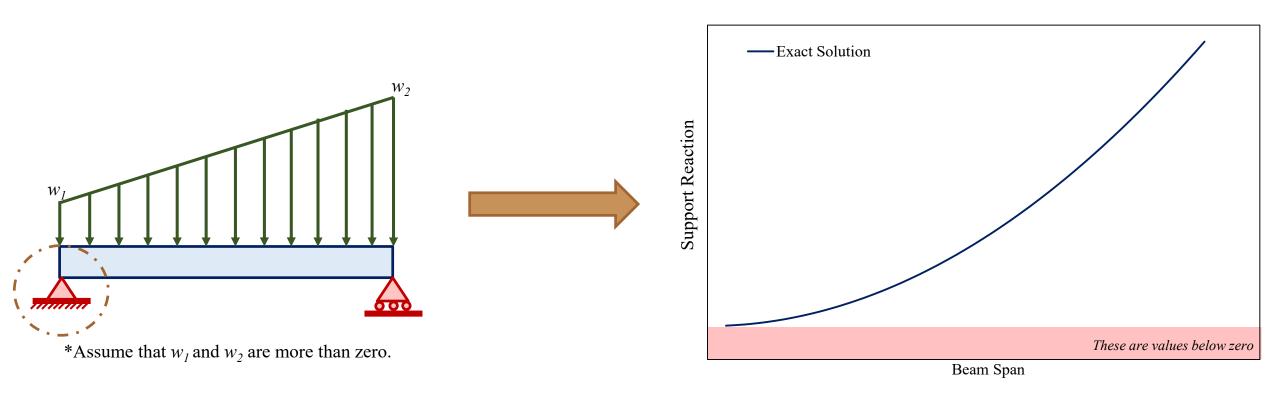


Figure 3: Illustration of the reaction estimation using the exact solution





















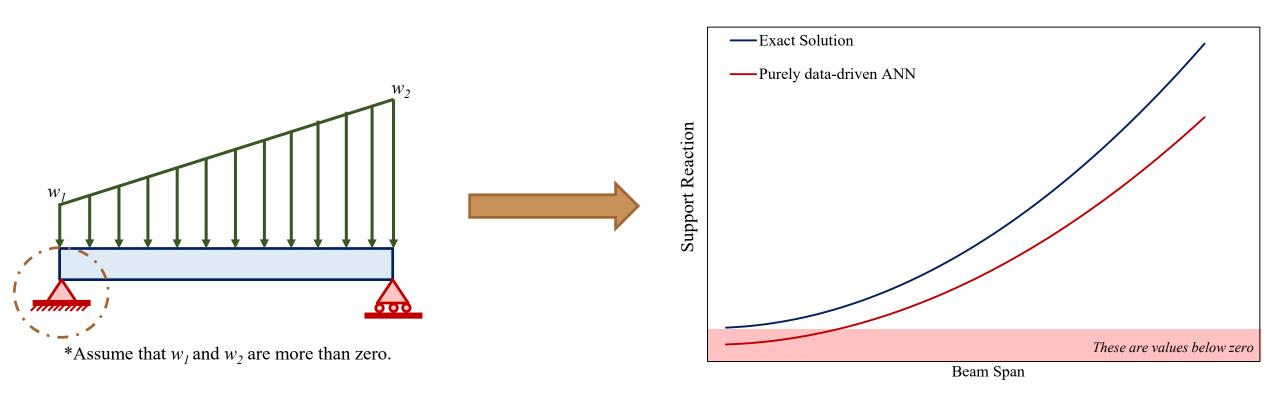


Figure 4: Illustration of the reaction estimation using various approaches



















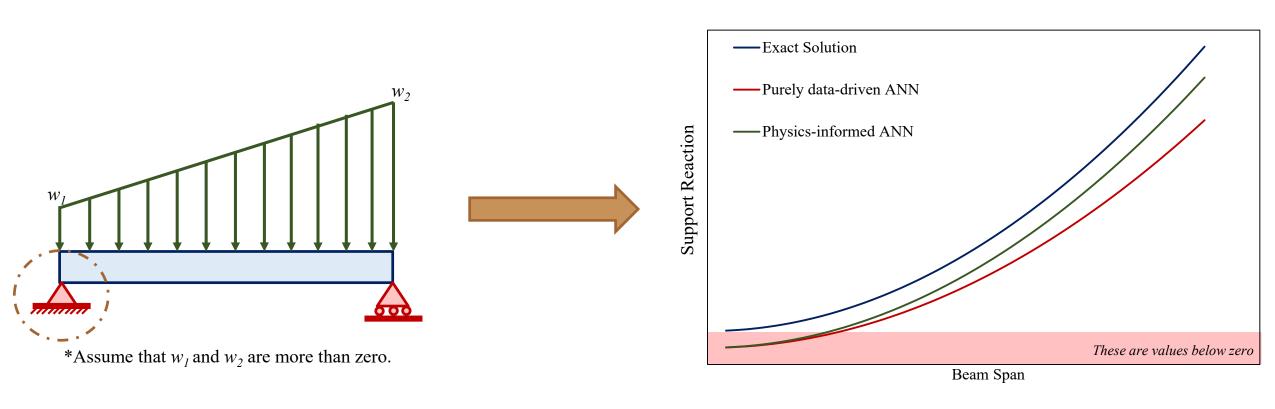


Figure 5: Illustration of the reaction estimation using various approaches

















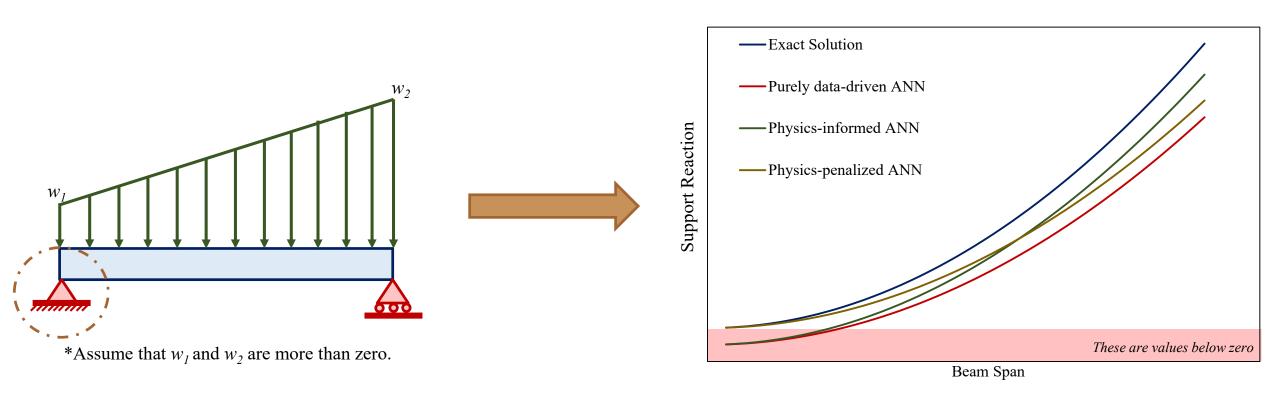


Figure 6: Illustration of the reaction estimation using various approaches















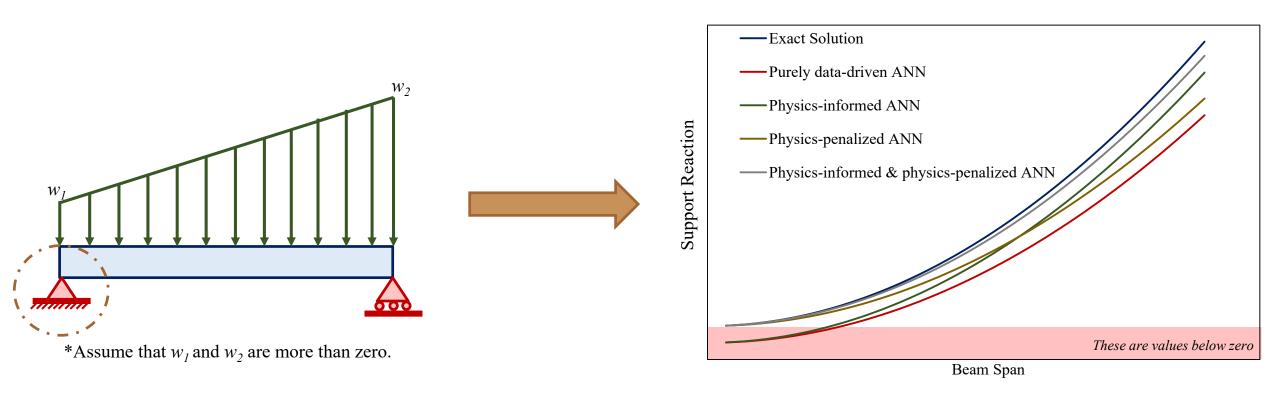


Figure 7: Illustration of the reaction estimation using various approaches









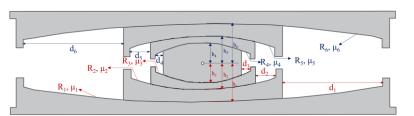








Preliminary Design of Quintuple Friction Pendulum Bearings



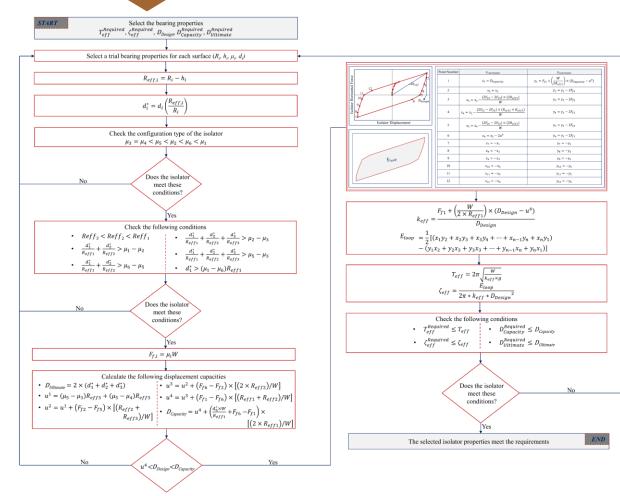


Figure 8: Conventional approach for preliminary design of quintuple friction pendulum bearings















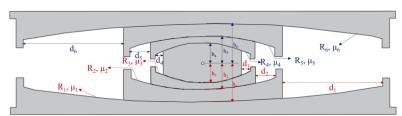








Preliminary Design of Quintuple Friction Pendulum Bearings



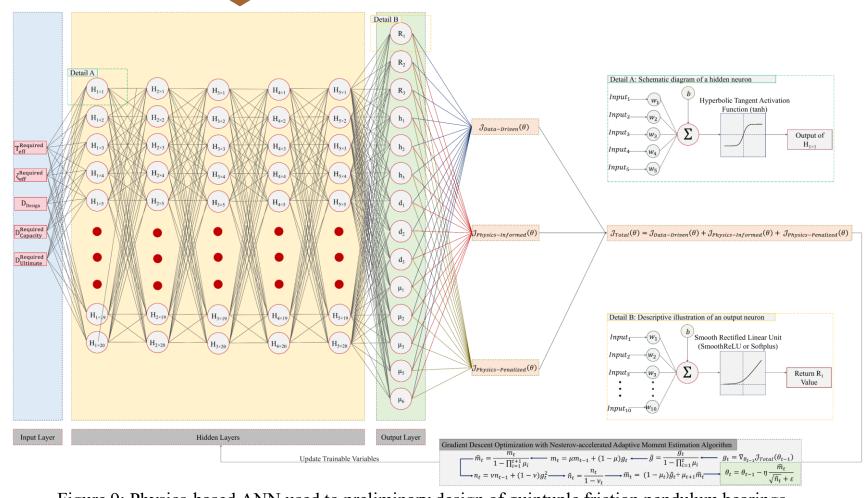


Figure 9: Physics-based ANN used to preliminary design of quintuple friction pendulum bearings













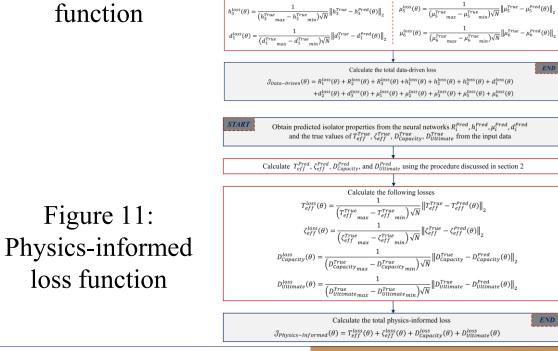








Figure 10: Datadriven loss



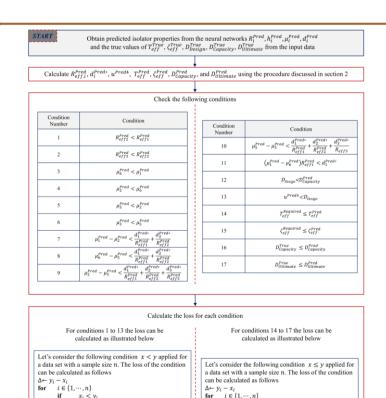


Figure 12: Physics-penalized loss function





Obtain predicted isolator properties from the neural networks R_i^{Pred} , h_i^{Pred} , μ_i^{Pred} , d_i^{Pred}

 $R_1^{loss}(\theta) = \frac{1}{(R_1^{True} - R_1^{True})\sqrt{N}} \left\| R_1^{True} - R_1^{Pred}(\theta) \right\|_2 \qquad d_2^{loss}(\theta) = \frac{1}{(d_2^{True} - d_2^{True})\sqrt{N}} \left\| d_2^{True} - d_2^{Pred}(\theta) \right\|_2$

 $R_{2}^{loss}(\theta) = \frac{1}{\left(R_{2}^{True} - - - R_{2}^{True} - R_{2}^{min}\right)\sqrt{N}} \left\|R_{2}^{True} - R_{2}^{Pred}(\theta)\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True} - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{Pred}(\theta)\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{Pred}(\theta)\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{True} - d_{3}^{True}\right\|_{2} \\ = \frac{1}{\left(d_{3}^{True} - - d_{3}^{True}\right)\sqrt{N}} \left\|d_{3}^{T$

 $R_{3}^{loss}(\theta) = \frac{1}{\left(R_{3}^{True} - R_{3}^{True} - R_{3}^{True} - R_{3}^{Pred}(\theta)\right)_{2}} \\ = \frac{1}{\left(\mu_{1}^{True} - \mu_{1}^{True} - \mu_{1}$

 $h_1^{loss}(\theta) = \frac{1}{\left(h_1^{True} - - h_1^{True} \right) \sqrt{N}} \left\| h_1^{True} - h_1^{Pred}(\theta) \right\|_2 \qquad \mu_2^{loss}(\theta) = \frac{1}{\left(\mu_2^{True} - \mu_2^{True} \right) \sqrt{N}} \left\| \mu_2^{True} - \mu_2^{Pred}(\theta) \right\|_2$

 $h_2^{loss}(\theta) = \frac{1}{\left(h_2^{True} - h_2^{True} \min\right)\sqrt{N}} \left\| h_2^{True} - h_2^{Pred}(\theta) \right\|_2 \\ \quad \left\| \mu_3^{loss}(\theta) = \frac{1}{\left(\mu_3^{True} - \mu_3^{True} \min\right)\sqrt{N}} \left\| \mu_3^{True} - \mu_3^{Pred}(\theta) \right\|_2 \\ \quad \left\| \mu_3^{loss}(\theta) - \frac{1}{\left(\mu_3^{True} - \mu_3^{True} \min\right)\sqrt{N}} \left\| \mu_3^{True} - \mu_3^{Pred}(\theta) \right\|_2 \\ \quad \left\| \mu_3^{loss}(\theta) - \frac{1}{\left(\mu_3^{True} - \mu_3^{True} \min\right)\sqrt{N}} \left\| \mu_3^{True} - \mu_3^{Pred}(\theta) \right\|_2 \\ \quad \left\| \mu_3^{loss}(\theta) - \frac{1}{\left(\mu_3^{True} - \mu_3^{True} \min\right)\sqrt{N}} \left\| \mu_3^{True} - \mu_3^{Pred}(\theta) \right\|_2 \\ \quad \left\| \mu_3^{loss}(\theta) - \frac{1}{\left(\mu_3^{True} - \mu_3^{True} - \mu_3^{True} - \mu_3^{True} \right)} \right\|_2 \\ \quad \left\| \mu_3^{True} - \mu_3^{True$















then $losses_i \leftarrow 0$

return $loss = \frac{1}{\sqrt{k}} ||losses||_2$

then $losses_i \leftarrow min(\Delta_i)$



Calculate the total physics-penalized loss
$$\begin{split} J_{Physics-Penelized}(\theta) &= Condition_1^{loss}(\theta) + Condition_2^{loss}(\theta) + Condition_3^{loss}(\theta) + Condition_4^{loss}(\theta) \\ &+ Condition_5^{loss}(\theta) + Condition_6^{loss}(\theta) + Condition_6^{loss}(\theta) + Condition_6^{loss}(\theta) \end{split}$$



 $x_i \leq y_i$

return $loss = \frac{1}{\sqrt{N}} ||losses||_2$

+ $Condition_{10}^{loss}(\theta)$ + $Condition_{10}^{loss}(\theta)$ + $Condition_{11}^{loss}(\theta)$ + $Condition_{12}^{loss}(\theta)$

+ $Condition_{13}^{loss}(\theta)$ + $Condition_{14}^{loss}(\theta)$ + $Condition_{15}^{loss}(\theta)$ + $Condition_{16}^{loss}(\theta)$

then $losses_i \leftarrow 0$ else if $x_i > y_i$

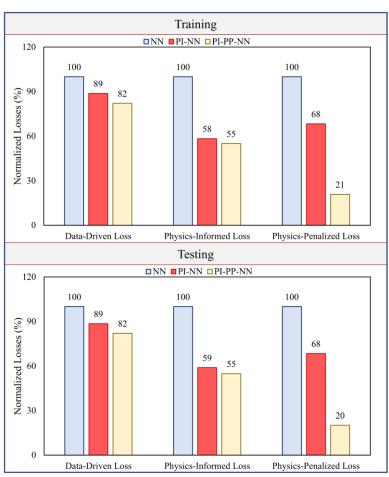


Figure 13: Comparison of different loss types for various ANN

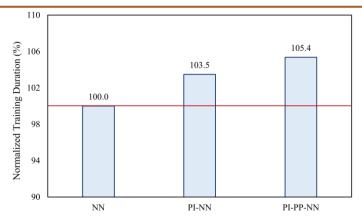


Figure 14: Comparison of training duration for various ANN

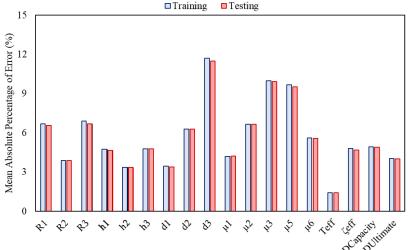


Figure 15: Accuracy of the physics-informed and physics-penalized ANN

















Nonlinear behavior of structures with viscous dampers (Zhang et al. 2020)

$$\mathbf{M\ddot{u}} + \underbrace{\mathbf{C\dot{u}} + \lambda \mathbf{Ku} + (1 - \lambda)\mathbf{Kr}}_{\mathbf{h}} = -\mathbf{M}\mathbf{\Gamma}a_{g}$$

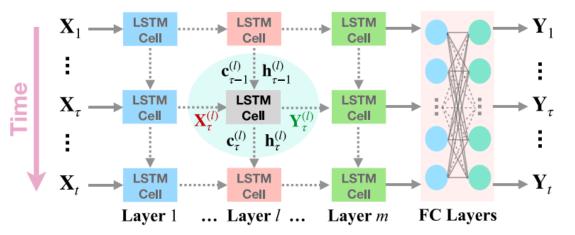


Figure 16: Schematic of the deep LSTM network used for prediction

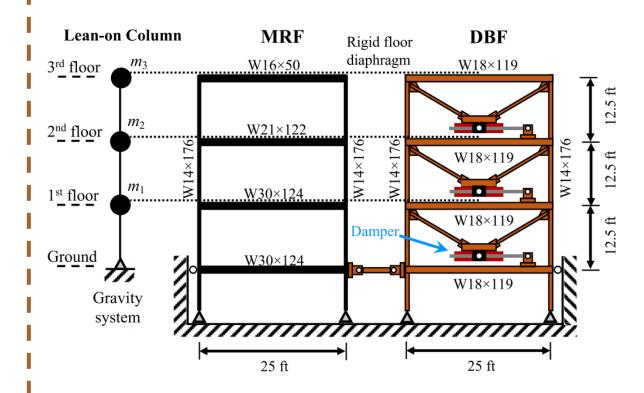


Figure 17: Illustration of the selected frame structure





















Nonlinear behavior of structures with viscous dampers (Zhang et al. 2020)

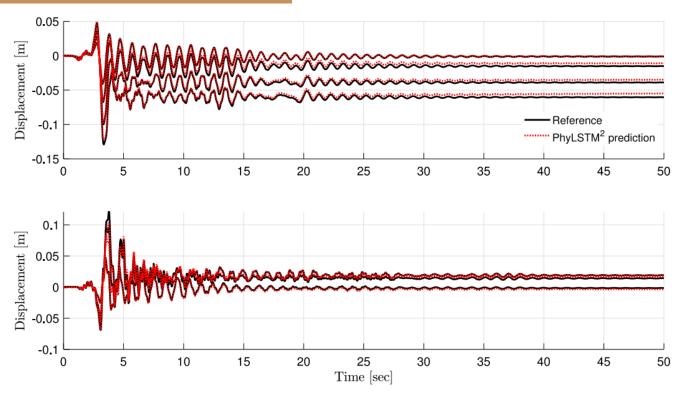


Figure 18: Predicted versus reference response of the building for two different earthquakes



















Response extrapolation (Lai et al. 2021)

$$\frac{d\mathbf{h}(t)}{dt} = f_{phy}(\mathbf{h}(t), t, \mathbf{u}(t)) + NN(\mathbf{h}(t), t, \boldsymbol{\theta})$$

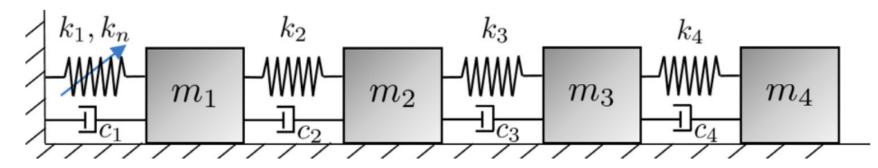


Figure 19: Illustration of a 4-DOF nonlinear structural dynamical system



















Response extrapolation (Lai et al. 2020)

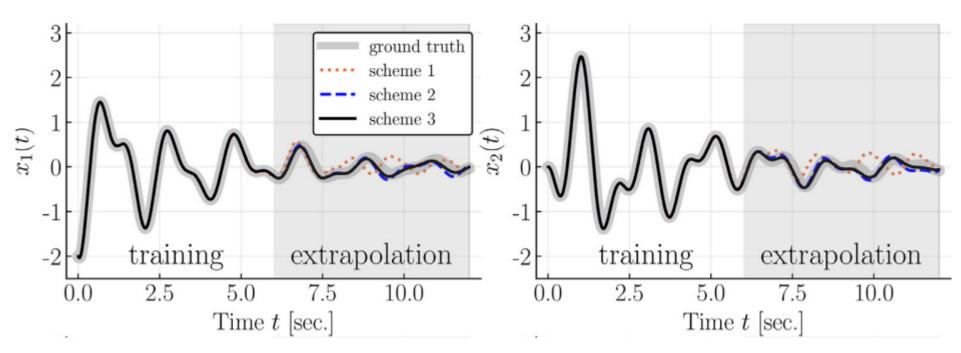


Figure 20: Response estimation using physics-informed ANN

















Conclusion

This study has focused on describing the PINN approach and briefly summarizing recent applications. On the basis of the aforementioned statements the following conclusions are drawn:

- PINN is still regarded as a new technology.
- PINN is a trending topic nowadays.
- PINN still hold many potentials in terms of control systems or complex modeling.



















ANY QUESTION

























REFERENCES

- Zhang, R., Liu, Y., & Sun, H. (2020). Physics-informed multi-LSTM networks for metamodeling of nonlinear structures. Computer Methods in Applied Mechanics and Engineering, 369, 113226.
- Lai, Z., Mylonas, C., Nagarajaiah, S., & Chatzi, E. (2021). Structural identification with physics-informed neural ordinary differential equations. Journal of Sound and Vibration, 508, 116196.

















